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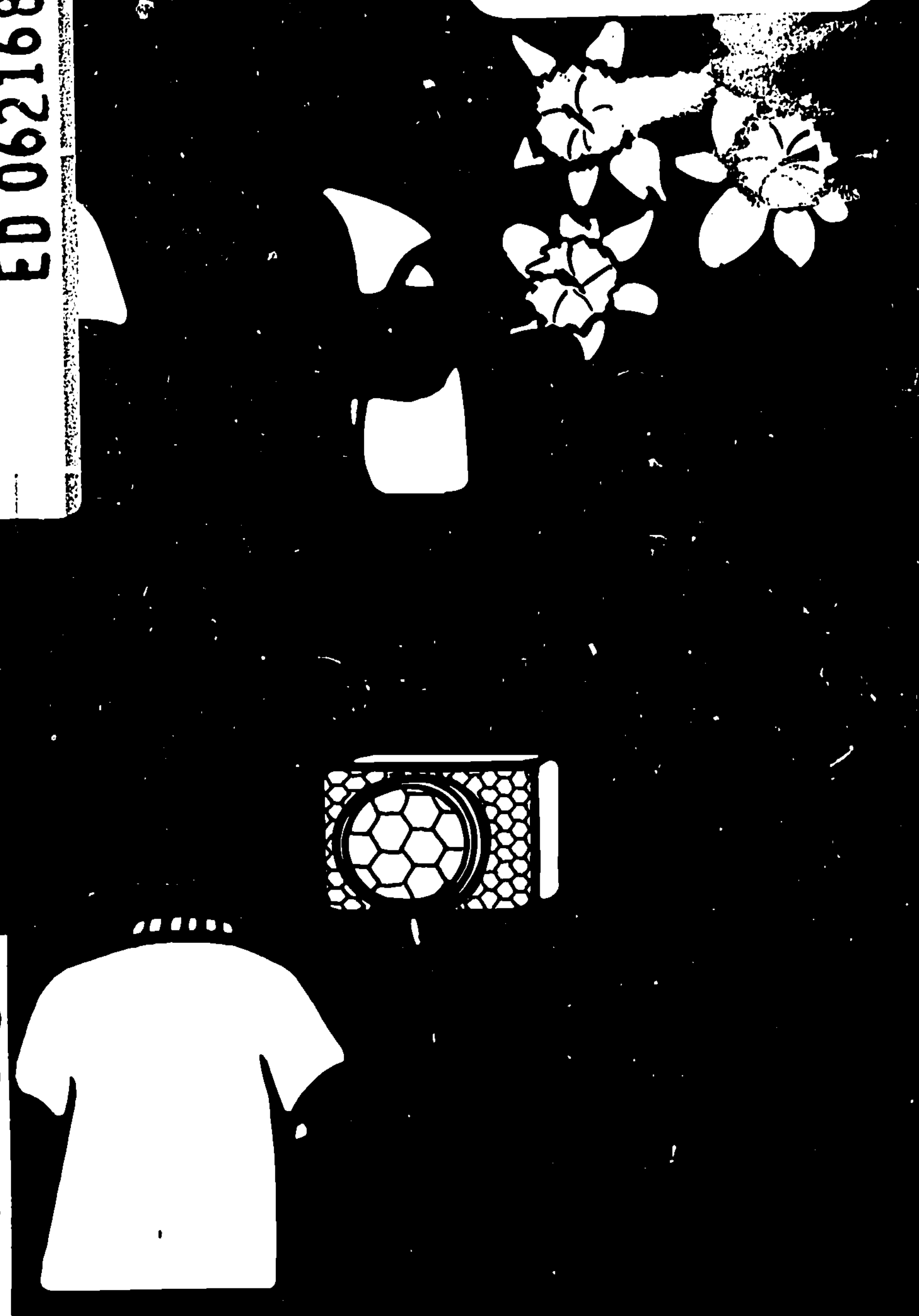
ABSTRACT

This booklet develops the theme that young children best learn mathematics through exploration of their environment. Several activities designed to promote the learning of concepts about space, numbers, and measurement are illustrated. Also discussed is the role of the teacher in sequencing instruction, selecting materials, and maintaining the appropriate emotional climate. (MM)

# MATHEMATICAL EXPERIENCING







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# MATHEMATICAL EXPERIENCING

by

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*The world is full of wonder.*
*There is much I wish to know.*
*Beauty rare I look upon—*
*Size and shape and pattern grow.*

# THE CHILD EXPLORES MATHEMATICS

# 1

The child's world is full of wonder. A large measure of this wonder is the sensing of mathematical ideas in each experience. The child's senses deliver a constant stream of mathematical thinking.

"My rock is bigger than yours."
"You have more than I do." (pieces of cake)
"The airplane goes higher, higher, higher. It
gets smaller, smaller, smaller, and . . ."
"My car goes faster than yours."

Mathematics is inherent in each experience, and mathematicking—i.e., reacting to the quantitative aspects of experiencing, is a *natural* activity.

*Exploration is the key to mathematical wonder.* It becomes an activity of reading the environment. It is a basic part of learning that suggests something of worth to be found. And, it implies unfulfilled curiosity and wonderment at the challenge of each new task.

Exploration may be likened to a journey. *The child explores his environment in response to his own curiosity. His ideas are personal creations that evolve from individual experiencing.* The child also explores the same environment under the careful guidance of his teacher, and the tour of exploration should be lined with specific, planned experiences. The planned experiences facilitate building systems of mathematical concepts. Eventually, as the child ventures further, he will explore with the tools of formal mathematics.

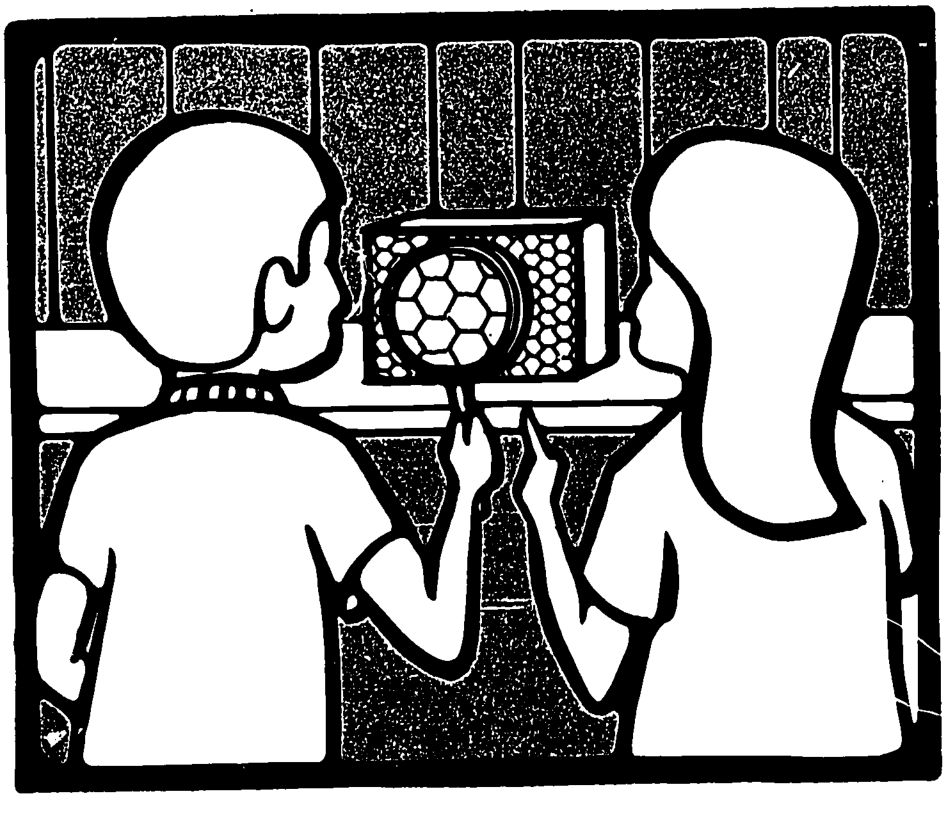
**The Child And His Environment**

*Exploring is a basic part of learning.* And, exploring is a natural activity for the young child. He explores the environmental things in his crib, his home, and his surroundings. A perceptive parent provides materials for exploring, such as empty boxes, cans, and other objects that are safe, clean, and appropriate. Often these playthings offer greater opportunity for desirable learning and fun for the child than many commercial toys.

In his activity, the child manipulates things to see how they work. He touches, feels, turns, throws, and experiments. He tries rolling things, pushing them, sliding them on the floor, banging them against the wall or a person. The child's explorations of his environment are very real to him, and they shed light on the learning that later becomes formalized. For example, the child uses his hand as an instrument to "measure" the inside of a box; he plays with blocks of various sizes and shapes, and he finds that cubes fit together to build a playhouse; he also fits together blocks making triangular patterns. His environment is rich in mathematical patterns, and he learns through wondering and finding out. As he senses ideas and makes discoveries, he formulates quantitative ideas.

*Children in the intermediate grades prosper through freedom to explore their own environment just as truly as younger children do.* For example, Jerry brought a honeycomb and a magnifying glass to his classroom. The children were excited to see the beauty in the pattern of the honeycomb. Each inspected the comb using the magnifying glass. The children noted that each cell was similar in shape. They discovered that the cells were hexagonal. Why do bees use a hexagonal pattern rather than a square or triangular one? Their wondering led to investigation and study.

*Honeycombs where bees put honey*
*Beauty of design I see.*
*Are cells a different size and shape?*
*They look so much alike to me.*



The learner's exploration of his environment should continue all of the days of his life. The exploration of the children in the intermediate grades should take on greater significance because of their previous experiences in the primary years. This exploring should produce ever increasing curiosity, appreciation, perceptiveness, and depth in procedures for investigating.

**The Child, His Teachers, And Peers**

There should be time for the learner to explore using materials and models that are provided by the teacher and pupils in the classroom. The exploring should be a continuation of the early environmental experiences of the child. *The greater the bond of continuity between outside and inside the classroom, the greater the opportunity for pupil growth.* The perceptive teacher strengthens this bond through being attuned to and knowing the child's background and activity.

Time spent in getting acquainted with the materials and seeing how they work pays off in later work for the child. He gets the feel of the materials through handling, turning, fitting together, taking apart, and finding new uses for them. The challenge of the peer group is activated as the children talk, ask questions, share discoveries, and develop procedures for finding answers to questions. This communal exploring serves the child well when he does more advanced problem solving at a later time. The materials become important *tools* for finding answers to mathematical questions—tools for problem solving.

**Selection and Use of Materials**

*The selection and use of materials is a very important part of teaching.* Materials should be selected to fit specific goals of instruction. *All* of the materials that are needed for the classroom *may be* things that the teacher and children bring into the room. Good commercial materials are available and may be used, but they should be selected wisely. Various materials should be in each classroom for general use, and those that fit the specific instructional program should be brought in when they are needed. These materials should serve to help pupils explore in a creative, personal, innovative way that leads to formulation of concepts and basic mathematical generalizations.

**Exploration Under the Guidance Of A Teacher**

There is a time for the child to explore his environment in response to his own direction and inquiry; there is also a time for him to see this same environment with new meaning under the direction of his teacher. Adult guidance should help him to evolve procedures for problem solving—efficient, sound ways of finding answers to mathematical questions, ways of measuring, skills for refining and communicating his discoveries. Using the tools of mathematics should challenge the learner to significant understanding rather than to *submergence* in a *sea* of *symbols* that demands an S.O.S.! Significance or submergence.*

Some of the greatest learning experiences occur fortuitiously. However, if all learning occurred this way, there would be no need for schools and school teachers. A good teacher does not leave the matter of learning to chance! He studies diligently to understand the nature of each of his learners, the nature of the subject that he teachers, and the evaluative procedures he may use to measure his handiwork in terms of pupil achievement.

Each learner is more complicated than the curriculum that is planned for him. A good teacher ever strives to know each child—to know

how he thinks

what he thinks

his background of concepts and skills

his strengths and his weaknesses

what releases him to his greatest potential

where he gets "hung up" with learning

his level of understanding of the specific things to be studied

where he is ready to go

To deal with the human intellect is an exciting yet demanding challenge. What makes the mind tick? What slows it down? What produces negative results? There is no text with answers in the back of the book. *Each person must be an explorer and a scholar if he is to earn the title,"teacher."*

Every subject has underlying fundamental *key concepts* that constitute its structure. The key concepts of mathematics are the basis of the curriculum. Selection of topics allotted to given mathematical levels of experiencing and identified behavioral objectives should be evolved by the teacher. These should be studied so that the teacher is released to plan pupil experiences that open the road to creative inquiry, exploration, study, and competence with mathematics. Curriculum planners, evaluators, mathematics educators, and others may help the teacher by organizing materials and courses of study. But, each teacher needs to personally conceive the goals of instruction, know its key concepts, and to realize the greatness and the simplicity,

*The term "or" is used in the exclusive sense of meaning that only one component is true.

order, pattern and beauty of the subject. The teacher can be given help in this accomplishment, but it must be a personal achievement that no one hands to him in packaged form.

Teaching and evaluation go hand-in-hand. New and improved procedures for evaluation need to be created. Sharper analysis of what the learner says, what he does, what he communicates, and what his attitude is toward mathematics needs to be given attention. Evaluation of the levels at which the learner recalls knowledge, comprehends ideas, analyzes by taking apart and fitting together main ideas, applies knowledge, and evaluates his own learning should be developed. Observable behavior may be evidenced in the laboratory setting in the classroom through group or individual discussion with other pupils and the teacher. Behavior is also observed through pupil response to problem-solving situations. Informal and formal testing using standardized, teacher-made, and pupil-created tests can he helpful, but evaluation of observed behavior is generally the most significant means. Written questions with written answers may give limited information, but directly observed behavior indicates a much broader basis for interpretation of how the learner solves problems.

### The Child Uses Tools of Mathematics

Problem-solving abilities are the *how to* tools for problem solving. There are many types of problem-solving situations that can be utilized to promote pupil growth in skills for problem solving. A pupil's efficient use of tools opens the door to making mathematics one of his favorite subjects. The tools free his mind from tedious, repetitive experiences. They help him to communicate and to be free to concentrate on problems that he wishes to solve. They are like power-driven instruments that help to get work accomplished.

Tools have little purpose as a thing in themselves. In some classrooms, the tools such as knowledge of number facts become the main goal of instruction while they should be considered only implements for developing concepts and generalizations. For example, consider tools such as the primary number facts and various shorthand language symbols such as $+, -, \div, \times, <, >, =$, and $-5$. These tools should be made meaningful to the learner, and they should be developed over a considerable period of time. To a great extent, recall should be a natural outgrowth that comes through use in problem-solving situations.

The facts and language symbols that need to be mastered by the learner should be identified by the teacher. For example, there are 100 primary facts for multiplication (base ten numeration). Nineteen (19) of these facts have 0 as one or both factors. Seventeen (17) additional facts have 1 as one or both factors. This makes a total of 36 facts that should be known to the learner through interpreting their meaning; no so-called drill should be needed. Additional analysis of the remaining 64 facts can simplify the task of teaching the primary facts. Most of the mastery of combinations can be a problem-solving activity if it is approaced in that manner. There is merit in helping the child see that he knows more of the facts than he doesn't know!

When facts such as those illustrated above are met in meaningful situations and are strengthened through using counters and other aids, many of the facts become mastered through immediate use. Later, when the learner faces more complete mastery, he can be helped to identify which of the facts he already knows and which ones he needs to master. The task of mastery becomes a simple one compared with drill exercises in which the learner uses a hit-or-miss technique that often brings frustration accompanied with dislike of what he conceives to be mathematics.

Various approaches to problem solving become tools in themselves. Some examples are:

- Having the child formulate problems that are meaningful to him followed with having him develop procedures for solving the problems that he created. The pupil who is released to create innovative rather than textbook-variety problems will make great strides in his skill.
- Making drawings, drawing diagrams, and making graphs for given problems that pertain to class interest. Use pupil-created and teacher-created problems that grow out of class activities.
- Selecting problems for which models will help illustrate a mathematical idea and having the child make his own models to illustrate the solution of the problem.
- Challenging the child to find a number of different ways to solve a given problem. Lead him to evaluate the efficiency of the various procedures.

# THE CHILD DISCOVERS MATHEMATICS

# 2

Subject matter is fused, integrated, and interrelated in experience. We identify and separate given subjects and topics so that we can look at them more penetratingly. *Three of the main topics of mathematics that are primary in the experiencing of children in nursery school through grade six are geometry, number, and measurement.* The topics are interrelated; they overlap in the thinking of a young child.

**The Discovery Of Space: Geometry (Spaceometry)**

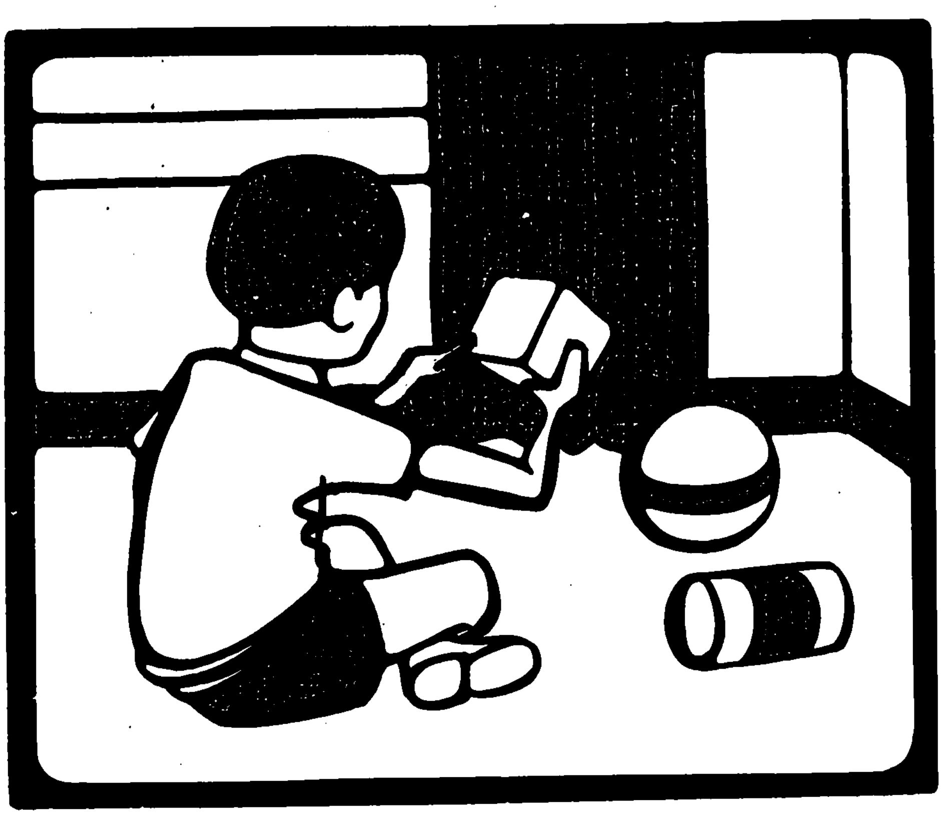
Today there is a great interest in the exploration, study, and measurement of space. The term spaceometry aptly identifies the subject. Spaceometry is a topic of natural interest to children, and it is being given increased consideration in today's program for mathematics in the elementary school.

It is likely true that the young child's knowledge of space is more advanced than his concept of number. The child explores space from before the time of his birth, and he enters school with considerable knowledge of space relationships. He is enveloped in a three-dimensional world, and some of his earliest responses are to his spacial environment. He intuitively senses, explores, studies, and "measures" his world of space. He thinks about location (whereness), direction, distance, shape, and so forth. He is a young mathematician who is excited with exploring his space universe.

Many of the child's early experiences include perceiving three-dimensional environmental models. Some of his first toys are balls (spheres), blocks (cubes), boxes (rectangular prisms), and cylindrical cans and containers. He feels their faces and their corners (if they have corners). He holds the objects in his hands and may run his fingers along the faces and the edges. He abstracts concepts of shape. Later, more precise models are used to refine his thinking about size and shape.

The solids offer a very good way to lead into two-dimensional figures. The ideas of two-dimensional figures may be a natural outgrowth of the child's feeling the faces, edges, and vertices of solids. Later, the child may trace around the faces of a cube or make a drawing of how a ball looks if he sees it as a flat figure. When such activity takes place, two-dimensional ideas are an extension of the three-dimensional ideas of the environmental models that he has held in his hands. The drawings are abstractions that he cannot hold in his hand the way that he did the solids. For example, the drawing of a circular region does not have the feel of the model of a sphere, and the square region does not have the feel of the cube (block).

When the child traces the faces of a cube, he sees that all of them are the same shape and same size (congruent). He may cut out one of the regions and fit it on the other regions of the given cube. When he traces the top and bottom of a right circular cylinder, he can see that the top and bottom regions are congruent.



Ideas of congruence develop early in the child's experiencing. He sees repeated patterns in tiles, textile designs, and patterns in nature such as the wings of butterflies. Teacher-guided discovery can build upon these through facilitating pupil-created symmetrical shapes, and through use of mirror images, paperfolding, papercutting, and tracing. The learner notes that the symmetry that one sees in a mirror is an example of congruence.

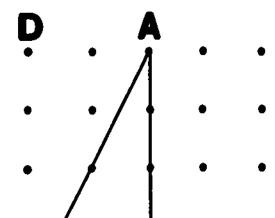
The child senses that some objects are similar to others in shape but are different in size. He is aware that a ball is similar in shape to an orange and that a sugar cube is similar to a large block (cube). He also sees the pattern of symmetry in the petals of a daisy. He may realize that congruence is a special kind of similarity—one in which the objects considered are not only alike in shape but also in size. Later, he uses the idea of scale to make maps and to create scale models. He may start with a cube and build other cubes to scale.

Only a few examples of the many different ideas of spaceometry are presented in this booklet. One of the newer considerations is that of motion geometry. The child enjoys sliding such things as his blocks and toy truck on the floor. He flies his airplane and sees how it looks when it flipped over and turned different positions. Later, he is excited with the ideas of making slides, turns, and flips, with pupil and/or teacher-created models.

Basic ideas of spaceometry that a child experiences in simple situations help him to interpret more difficult examples at a later time. The child of intermediate grade level builds upon previous experiences. An example of this is the development of the formula for the area of a triangular region.

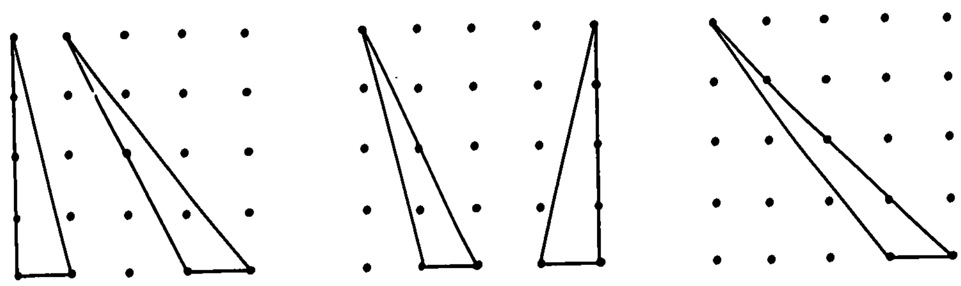
Previous learnings include the ability to find the area of a triangular region constructed on a geoboard. For example, the area of the region ABC can be found by using the following ideas:

(1) the area of the rectangular region ABCD is equal to eight square units (count them);
(2) the two triangular regions ABC and CDA are congruent (hence their areas are the same); and
(3) the area of triangular region ABC must be one-half that of the region ABCD (hence its area is four square units).



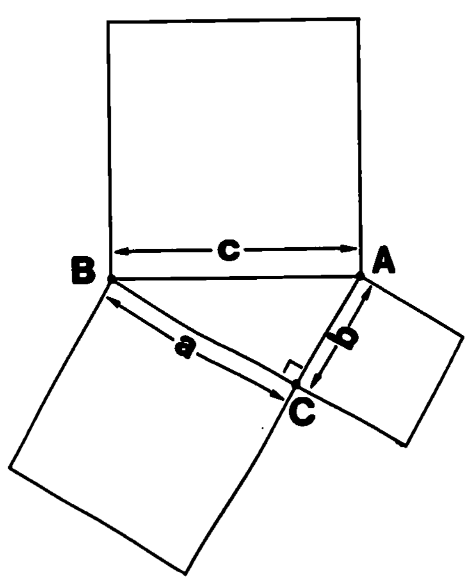

Areas of other triangular regions may be found through the process of adding and/or subtracting regions found as in the above example.

With these learnings, a teacher might ask his children to construct the following series of triangles on their geoboards:



What is the area of each of the triangular regions? What is constant for each? What varies? Answers to these and other similar questions should lead the children to discover that the base and the altitude of a triangle are related to the area of its region. Further refinement of this "discovery" may lead to the suggestion for the formula for the area of a triangular region.

As another example, two children are seen to be wrestling with a problem first encountered by the ancients. "If a square be constructed on the hypotenuse . . . then the sum of the squares . . . is equivalent to the square on the hypotenuse."



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One child may think, "How can I cut these smaller squares so that they will fit on top of the larger square?" Perhaps between the two children, a method will be suggested from previous experience which involves the partitioning of the smaller squares to make a congruent "covering" for the larger. Or, the teacher may suggest a means to do this. Having successfully completed this task, will the technique hold for all right triangles? Without construction of squares, how might we determine whether the relationship holds? These questions should lead naturally to the familiar equation, $a^2 + b^2 = c^2$, for all right triangles, $c$ being the length of the hypotenuse and $a$, $b$, the lengths of the other sides. With the geometric idea generalized to an equation, the two students may then go about verifying its validity by direct measurement and computation.

Through these and other examples, we find that children are introduced to geometry and more broadly, spaceometry. The type of mathematics suggested by spaceometry not only serves the child but prepares him for his own unique contribution to society.

**In Pursuit Of How Many: Number**

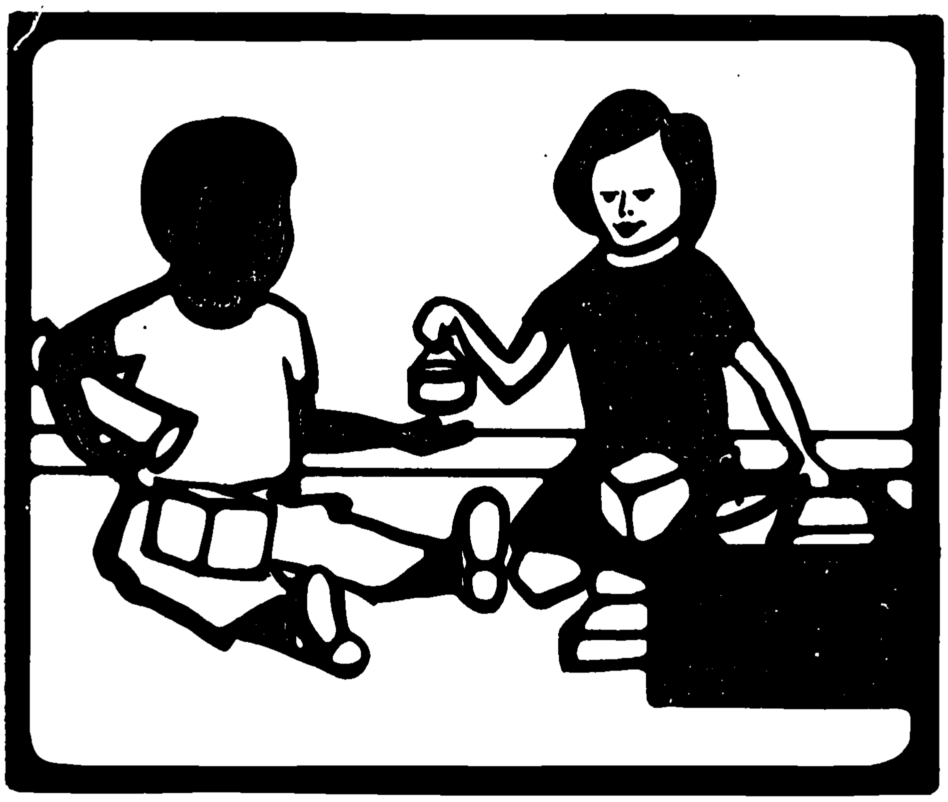
The ideas and language of number develop in a stream of mathematical thinking that a child experiences. Number concepts help the child refine and communicate his ideas of quantity and quantitative relationships. These ideas grow out of situations for which the notion of "how many" emerges. A very young child is aware of "how manyness" in an unstructured sense. He notes the presence or the absence of something that he wishes to have—or not to have. The ideas of "not any" (an empty set) and "some" begin and develop early in his perception of his world. The child perceives that two given sets are numerically equivalent or that they are not numerically equivalent. Later he identifies which of two sets has the greater number of elements and which has the lesser number.

Mathematical experiencing begins and expands in a natural way, but competence is rarely achieved without the guidance of a parent or other teacher. Examples such as the following illustrate this type of experiencing and its facilitation by others who guide the child.

*One-to-one correspondence.*—The child sees that there is a one-to-one correspondence between the number of children and the number of oranges that are on a tray. His visual perception gives evidence to the fact that there is one orange for each child. He may also match sets of blue-colored blocks with red-colored blocks and see that he has as many of one set as of the other.

In another circumstance, there are not enough tricycles for each child to ride at the same time. The child is keenly aware that there is not a one-to-one correspondence of children and tricycles.

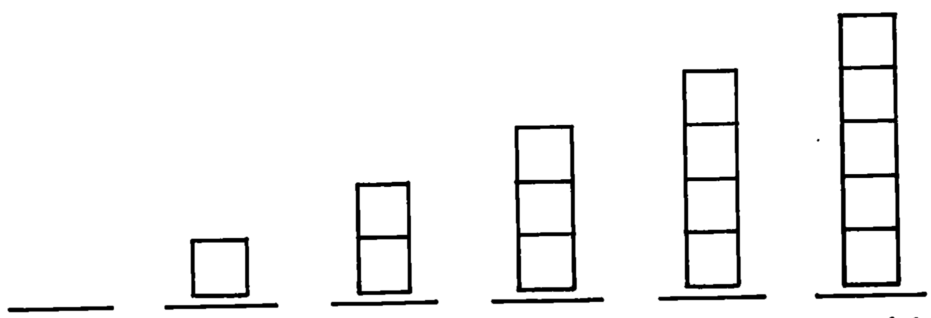
*Early ideas of division.*—A treasury of many objects has been discovered by two children. The children are dividing: "You take one, I take one, you take one, I take one . . ." Thus, this division discussion may be one of the earliest ideas that a child experiences.



*Names for numbers.*—Early ideas of numbers are usually tied with specific things. The child thinks of 4 toys, 4 oranges, 3 ribbons and so forth. Later, the number property is abstracted, and the ideas become freed from specific things. The child sees that number is not related to the concrete things that are counted, but rather it answers the question, "How many?"

*Ideas of a number scale.*—The idea of a number scale is initiated when an identity (a unit) is selected. The absence of the identity suggests "not any" (an empty set). The presence of the unit is the simplest amount that can be conceived. By adding one more unit to form a new group and continuing the procedure to form additional new groups, the idea of a scaled series emerges. Consider blocks or other stackable objects arranged in a pattern.

Use one block as a unit and add one more block to each successive group so that each new group contains one block more than the preceding group. The learner readily perceives that each newly-made group has one block more than the preceding group. He also senses



that each new group has one block less than the group which follows it.

Much later, the learner will be challenged to see that one may select any given set of objects to start (an origin) and use that set as a referent set for making comparisons. These simple ideas of origin and direction become powerful in the child's thinking. The ideas that are formulated with the positive whole numbers can be so meaningful that they are freed for interpretation later with the integers and the creation of coordinate charts.

*Counting: one fundamental operation.*—Counting is the one fundamental operation with number. The basic operation is change. Change can occur as *increase* or as *decrease* of quantity. The two directions are opposite in function, and the understanding of each of them is necessary for intelligent understanding of either of them. Addition counts *into*; it expresses increase of quantitative value. Subtraction counts *out of*; it expresses decrease of quantitative value. When additions occur successively with equal values used as addends, the operation may be shortened to multiplication. Similarly, with examples for which subtractions occur successively with subtrahends of equal values, division may be treated as a special case of subtraction. For example, consider the example, $4 + 4 + 4 = 12$.

$$\begin{array}{r} 4 \\ 4 \\ +\ 4 \\ \hline 12 \end{array} \qquad \begin{array}{r} 4 \\ \times\ 3 \\ \hline 12 \end{array} \qquad \begin{array}{rl} 12 & \\ -\ 4 & (1) \\ \hline 8 & \\ -\ 4 & (2) \\ \hline 4 & \\ -\ 4 & (3) \\ \hline 0 & \end{array} \qquad \begin{array}{r} 3 \\ 4\overline{)\ 12} \\ 12 \\ \hline \end{array}$$

There are many ideas of one fundamental operation, and the examples above illustrate some of the related ideas of addition, subtraction, multiplication, and division.

There are many concepts of number and numeration systems that are important for the child to understand and to interpret. Included among the ideas of positional numeration are the ideas of: (1) the constant (or face) value represented by a numeral, (2) the value of the place a numeral holds (powers of ten in the decimal system), and (3) the positional (or place) value represented by a numeral (the value of the place it holds multiplied by its constant value). Thus the constant value of the "3" in the decimal numeral 6,302 is "three," the value of its place, "one hundred" ($10^2$) and its positional value, "three hundred."

The zero has some unique characteristics as used in the system of numeration. As a single number, it designates the origin on the number line, conventionally negative numbers being to the left of it and positive numbers being to the right. It may also serve as a placeholder in its own right as is true for each digit, but zero represents "not any" as both its constant and positional value. It may also suggest precision in measurement such as .30 where the "zero" tells us that we have a measure of $^{30}/_{100}$ rather than one of $^{3}/_{10}$. Of course, the zero serves a unique function as the identity element for addition so that we may add it to any real number and not change the value of the sum.

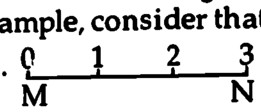
If one has a good understanding of the decimal system, he can readily interpret bases other than ten. Limited instruction in another base may be helpful in developing the ideas of our positional system and determining how well a child understands the concepts of the decimal system. Computational feats using bases other than ten are of questionable value.

Numerical computation is facilitated through understanding of the relationships, properties, and concepts that underlie computations. Binary operations, identity elements, the consideration of such things as divisor-dividend-quotient relationships and factor-product relationships, and the commutative, associative, and distributive properties are important. For example, in the multiplication example, 5 x 2 x 9, the child may find that associating (5 x 2) x 9 is easier for him than 5 x (2 x 9) since he may be more familiar with 10 x 9 than with 5 x 18.

One of the most important considerations for the teacher at all grade levels is to free the learner to achieve a high level of appreciation and of interpretation of the numerical systems that he studies. The basic concepts combined with sharpness of identifying and of using efficient procedures for mastering basic number facts can help to lead a child to a desirable level of mastery of facts.

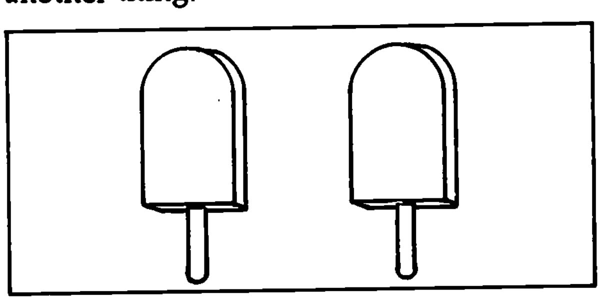
**The Descriptive Quest: Measurement**

Measurement for the elementary school pupil is a process of learning how to represent quantities in an organized fashion. Learning measurement may be thought of as both a *process* and a *product.*

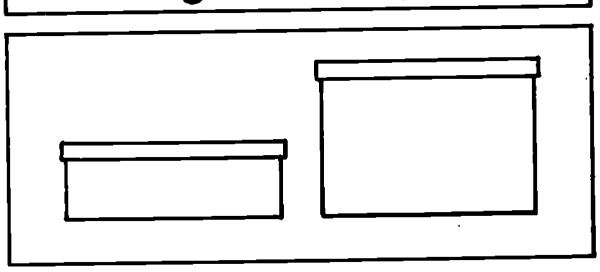
Mathematically, we associate a real number with a physical observation of some property such as length, weight, area, volume, and velocity.* The *product* of this association is a *number* along with a suitable unit that is used as a measure. For example, consider that the length of line segment MN is 3 centimeters. 

The measure of MN is 3, and the unit of measure is *centimeters*. The measurement of MN is *3 centimeters*.

How does the child make accurate measurements and represent physical observations in a practical way? This question leads one to look at measurement as a *process*. A child left to his own devices soon becomes involved in informal measuring experiences. He notes properties of things and perceives *likenesses* and *differences*. The process is simple and intuitive. The child notes that the length of one thing is longer than, shorter than, or the same length as that of another thing.



The length of Billy's and Susan's ice cream bars is the same



The height of Joe's box is 2 times as great as that of Tom's box.

In the early stages of making comparisons, if it is important to the child, he may wonder how much the property of the two things varies.

Comparison of a physical property requires the use of a *referent*. The child compares a particular property of one thing with the same property of something else. For example, a young boy observes a motorcycle go by. In attempting to "catch up" on his tricycle, he soon perceives that he cannot go as fast as the motorcycle. His speed becomes the referent, and the speed of the rapidly disappearing motorcycle is compared with it. This same boy is shorter than his father. His father's height becomes the referent in this case.

*Measurement as noted, is restrictive but defines the subject for elementary school experiences.

Simple comparisons usually involve the ideas of an *origin* for measuring and the *direction* of variation from the origin. Consider the example above of the young boy comparing his height with that of his father. The origin for determining that he was shorter than his father was the level place where they were standing. Since he used his father's height as a *referent* and related his height with it, the direction of variation was that of decrease—i.e., he was shorter than his father. The idea of difference (variation) involves the awareness of an origin from which the variation occurs. With these and other experiences, this youngster arrives in kindergarten having already developed many of the ideas of measurement as a process.

In teaching measurement, the objective of the teacher is to guide his children through varied experiences to maturity of thought. The children are taught to use a unit of measure as a referent and re-use it over and over again in making comparisons of physical properties. They are taught to select suitable referents for differing purposes.

The *number line* is introduced, and the idea of a *point of origin* soon takes on greater significance. For instance, the measurement of temperature lends itself to the use of a number line, a convenient point or origin being the location on the scale at which water freezes. *Directionality* is developed as the child talks about temperatures that are above or below the freezing point. Where the metric system is used, the point of origin coincides with the zero on the number line. A point of origin need not coincide with the zero, but it is certainly more convenient if it does.

Arbitrary points of origin are selected intuitively by the child or with help from his teacher. He may not be able to verbalize his discovery in terms common to the mathematician, however. Directionality is spontaneous in a child's frame of reference. The thing to be compared (relatum) is larger than or smaller than the referent. Taken compositely, the development of these ideas contributes to maturity in the process of measurement.

What should a teacher do to facilitate the process of learning to measure? Perception is enhanced by the teacher who challenges the learner to note things that may be compared. Encouraging children to feel and hold objects establishes a basis for the perception of weight and volume. Feeling hot and cold objects such as a black painted can left in the sun and a tray of ice-cubes suggests ideas of temperature and heat. The teacher should seek to structure many activities which introduce children to quantitative ideas such as temperature, length, time, shape, texture, color, light intensity, area, weight and volume.

The teacher helps a child form accurate ideas of measurement by first having him use non-standard referents. The teacher may challenge the children to develop the idea that parts of the body serve this purpose very well. The children may measure the width of their room with their feet or by strides. Desks are measured using fingers and palms or other parts of the hand. The playground is paced off. Other activities will suggest themselves as the teacher pursues this line of thought in his teaching.

Teaching non-standardized units should be followed by the teaching of standardized units of measure. Children may be introduced to these units in response to their own desire to communicate their measures to their classmates.

What sorts of materials will enhance the process of measurement as it has been discussed? What equipment should a teacher have to teach measurement? The following is a selected list of materials which can be easily obtained or constructed by the teacher for use in teaching the process of measurement:

- capacity containers of various sorts, both standard and non-standard
- metric measures as well as English
- graph paper for records and scaling
- homemade balances, postal scales and other weight measuring instruments
- clock faces, calendars, and materials to construct clock faces
- pictures of measuring instruments
- pupil-made micrometers
- small ceramic tiles to use for non-standard area measure
- geoboards for development of simple measurement and mensuration ideas
- strips of paper for making class graphs or use as non-standard referents

Many commercial aids are available but it may be found that maximum value is obtained from the devices and materials which are teacher and/or pupil collected (or constructed).

What are some examples of the teaching which is suggested in this discussion? A primary grade teacher may open a discussion by asking his children how far it is across the room. He accepts answers but suggests that the class decide how they may find out. Pacing or body measures such as "feet" may be suggested as a means to find the answer. Assuming that "feet" have been suggested, two children are asked to measure the distance. The teacher may also measure the distance using his own "feet." Recording what is found and deciding what to do about any differences sets the stage for development of the idea that all should agree on the "foot" to be used for measuring. This local "standard" foot may then be reproduced by drawing around it and the children led to use it in other measurements.

An intermediate grade teacher may give each child a worksheet with several polygonal shapes drawn on it. An additional sheet may be given on which free-form closed curves are drawn. With these sheets, the children are given a collection of tiles or pieces of paper. The tiles and paper should include sets of congruent shapes. (e.g., varied shapes, squares, rectangles and triangles.) The teacher may also use an overhead projector on which the same or similar polygonal and free-form curves are sketched. The teacher then poses the following question: "How might we 'cover' the shapes we have on our worksheets?" Accepting all answers, he records the results. With many "standards" being used it becomes important to agree on one for desirable communication. Eventual introduction of the standard "square" measure for area may follow lessons such as this one.

Measurement, one of the most important topics for children of the elementary school includes measuring weight, time, distance, capacity, and volume. It should lead to the child's developing formulas for areas, volume, and capacity. Knowledge and skills with standard units of measure are basic for facile communication.

The learner should also be introduced to the metric system. Appreciation for its simplicity includes familiarity with the meter (length), the gram (weight), the liter (capacity), among others. The study of the metric system provides opportunity for children to see how such measures as those of weight, length, and capacity may be decimally interrelated. For example, one liter of pure water (under specified conditions) weighs one kilogram and will fill a cube-shaped container, an edge of which is 10 centimeters long. Also, the various units of the metric system are decimal multiples and submultiples of the basic units, which facilitates computational procedures.

Study of measurement is exciting in its variety, interest, and practicality. An otherwise dull presentation of number may be enhanced by the wide diversity of measurement topics. Measurement appeals to the child and the adult-to-be in like fashion, being a practical outcome of the mathematics program. Attitudes, techniques and knowledge are all brought to bear in the study of measurement, and its application cuts across many lines of study.

# THE QUEST OF THE TEACHER

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Each good teacher realizes that he is a very important person who should strive to become increasingly important to his profession. He realizes that pupil growth is determined to a great extent by the ability of the teacher. He knows that he creates the environment for learning. He sets the stage for his pupils to be freed to learn—freed to enjoy achieving continuously higher levels of competence. The freedom to learn releases the child to be inspired to think for himself. He explores and discovers on his own. He may go in a very different direction toward the goal that the teacher had planned, and the result may be very rewarding.

Many factors underlie a teacher's success in teaching mathematics. One of the very significant factors is the teacher's understandings and appreciations of the concepts and patterns of mathematics. The background for this cannot be measured merely by the number of courses that the teacher has completed, but courses should be helpful and should be required. Understanding and appreciation are personal achievements that the teacher should strive to grow toward as long as he lives, but especially so during his professional life.

The understandings and appreciations of the teacher facilitate his seeing the program for mathematics as an organized whole. It helps him to see how the parts fit together and to know what comes before and what comes after a given instructional unit. It leads to his being perceptive regarding the specific teaching that he plans for his class. It helps him evaluate the learning that takes place in his classroom. It challenges him to see and to interpret the mathematics that permeates the child's activities. It gives him freedom to teach because he has confidence in himself. It releases fear that is often present for teachers who are afraid to teach and hold class by going through the textbook a page at a time.

The following examples are some of the important things that a teacher does in his quest for making mathematics one of the more interesting and profitable subjects that he teaches.

- The teacher becomes increasingly aware of the child's wonder of mathematics within his environment. He develops teaching procedures that make wonder continue to grow.

- *The teacher creates an emotional environment that releases pupils to freedom to learn through experimentation and exploration.* He sets aside periods in which the learner can experiment, think, direct his own activity, and formulate his own problems and offer solutions to his problems.

The teacher sees that when small ideas are expressed and freed, they often grow into big ideas.

The teacher helps the child build new interests that blossom into higher levels of mathematical learning and understanding.

The teacher understands the dynamics of pupil discovery and applies them through procedures such as the following:

- Appreciates each child's problem-solving activities and frees the child to tell his idea in his own words. He listens to what the child says, lets questions come from him, and analyzes his procedures for solving problems.
- The teacher continuously evaluates pupil learning that involves the assessment of and interpretation of ideas, generalizations, concepts, numerical computation, abilities, and problem-solving skills.

The exploration and study that the pupil has achieved go with him as he meets higher levels of mathematical thinking in geometry, algebra, the calculus, and other advanced study. His experiences in the elementary school lead him far beyond the point his teachers envisioned for his early years. *To challenge the learner to stretch and to achieve a high level of mathematical competence is the goal toward which each teacher reaches.*

As each teacher observes within his learners the growth of wonder in mathematical ideas, he finds excitement and delight in teaching. He learns from each child that he teaches and is released to see with fresh, new vision, because he, too, grows through continued mathematical experiencing.

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During the summer of 1971, she was on the faculty of the International Seminar for Advanced Education in Curriculum Development and Innovation that was held in Grånna, Sweden. Professor Benjamin S. Bloom was the Director of the Seminar, and twenty-four different countries sent teams of four to six participants to the Seminar.

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